Period 1, Sept 30, 2024

4. $y = \tan^2(3\theta)$ $\Rightarrow y = \tan^2 u \Rightarrow y = L^2$

$u = 3\theta$ $\quad L = \tan u$ $\quad \frac{dy}{dL} = 2L$

$\frac{du}{d\theta} = 3$ $\quad \frac{dL}{du} = \sec^2 u$

$\frac{\cancel{du}}{d\theta} \cdot \frac{\cancel{dL}}{\cancel{du}} \cdot \frac{dy}{\cancel{dL}}$

$3 \cdot \sec^2 u \cdot 2L$

$6\sec^2 3\theta \cdot \tan u$

$6\sec^2 3\theta \tan 3\theta = \frac{dy}{d\theta}$

5. $y = \sin(\cos x)$ at $x = \frac{\pi}{6}$;

$y = \sin u$

$u = \cos x$ $\quad \frac{dy}{du} = \cos u$

$\frac{du}{dx} = -\sin x$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}}$

$-\sin x \cdot \cos u$

$-\sin x \cos(\cos x) = \frac{dy}{dx}$

$-\sin\frac{\pi}{6} \cos(\cos\frac{\pi}{6}) = \frac{dy}{dx}$

$- \cdot \frac{1}{2} \cdot \cos\left(\frac{\sqrt{3}}{2}\right) = \frac{dy}{dx}$

3. $h(t) = \sqrt[3]{2t+7} + e^{t^2}$

$y = \sqrt[3]{2T+7} \Rightarrow y = \sqrt[3]{u}$

$u = 2T+7 \Rightarrow y = u^{\frac{1}{3}}$

$\frac{du}{dT} = 2$ $\quad \frac{dy}{du} = \frac{1}{3u^{2/3}} = \frac{1}{3\sqrt[3]{u^2}}$

$\frac{\cancel{du}}{dT} \cdot \frac{dy}{\cancel{du}}$

$2 \cdot \frac{1}{3\sqrt[3]{u^2}} = \frac{2}{3\sqrt[3]{(2T+7)^2}}$

$y = e^{T^2} \Rightarrow y = e^u$

$u = T^2$ $\quad \frac{dy}{du} = e^u \cdot 1$

$\frac{du}{dT} = 2T$

$\frac{\cancel{du}}{dT} \cdot \frac{dy}{\cancel{du}}$

$2T \cdot e^u = 2Te^{T^2}$

$h'(T) = \frac{2}{3\sqrt[3]{(2T+7)^2}} + 2Te^{T^2}$

6. Let $r(x) = f(h(x))$, where $h(2) = 3, h'(2) = 4, f'(2) = 2, f(2) = 1, f'(3) = 6$, find $r'(2)$.

$r(x) = F(h(x))$

$r'(x) = F'(h(x)) \cdot h'(x)$

$r'(2) = F'(\underbrace{h(2)}_{3}) \cdot \underbrace{h'(2)}_{4}$

$r'(2) = F'(3) \cdot 4$

$r'(2) = 6 \cdot 4$

10. (Challenge/Optional) Find the derivative of $y = cos^2 \sqrt{\sin(\tan(\pi x))}$

$u = \pi x$

$\frac{du}{dx} = \pi$

$y = \cos^2 \sqrt{\sin(\tan u)} \Rightarrow y = \cos^2 \sqrt{\sin L} \Rightarrow y = \cos^2 \sqrt{n} \Rightarrow y = \cos^2 r$

$L = \tan u$

$\frac{dL}{du} = \sec^2 u$

$n = \sin L$

$\frac{dn}{dL} = \cos L$

$r = \sqrt{n} = n^{\frac{1}{2}}$

$\frac{dr}{dn} = \frac{1}{2\sqrt{n}}$

$J = \cos r$

$\frac{dJ}{dr} = -\sin r$

$y = J^2$

$\frac{dy}{dJ} = 2J$

$\frac{d\cancel{u}}{dx} \cdot \frac{d\cancel{L}}{d\cancel{u}} \cdot \frac{d\cancel{n}}{d\cancel{L}} \cdot \frac{d\cancel{r}}{d\cancel{n}} \cdot \frac{d\cancel{J}}{d\cancel{r}} \cdot \frac{dy}{d\cancel{J}} = \frac{dy}{dx}$

$\pi \cdot \sec^2 u \cdot \cos L \cdot \frac{1}{2\sqrt{n}} \cdot -\sin r \cdot 2J = \frac{dy}{dx}$

$\pi \cdot \sec^2 \pi x \cdot \cos(\tan \pi x) \cdot \frac{1}{2\sqrt{\sin(\tan \pi x)}} \cdot -\sin\left(\sqrt{\sin(\tan \pi x)}\right) \cdot 2\cos\left(\sqrt{\sin(\tan \pi x)}\right)$

$$\text{Position} = S(T) = \frac{1}{2}aT^2 + V_0 T + S_0$$

initial velocity (starting velocity)

acc = gravity Earth $-32\ \text{feet}/\text{sec}^2$

initial Position

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{\text{Change in } y}{\text{Change in } x}$$

$$y = \frac{1}{2}aX^2 + V_0 X + S_0 = \text{position}$$

constants (a, V_0, S_0)

$$\frac{dy}{dx} = \frac{1}{2}a \cdot 2X^1 + V_0 \cdot 1X^0 + 0 = \frac{\text{Change of Position}}{\text{Change of } X} = \frac{\text{Change of Position}}{\text{Change of Time}}$$

$= \text{Velocity}$

$$\frac{dy}{dx} = aX + V_0$$

$$\frac{dy}{dT} = \text{velocity}$$

$$\frac{d^2y}{dx^2} = a \cdot 1X^0 + 0 = a = \frac{\text{Change in velocity}}{\text{Change in Time}} = \text{acceleration}$$

$S(T) = \text{Position}$

$S'(T) = \text{Velocity} = V(T)$

$S''(T) = V'(T) = \text{acceleration} = a(T)$

at rest $V(T) = 0$

Change of direction

$V(T)$ goes from $+$ To $-$ or From $-$ To $+$

Speeding up = when $a(T)$ and $V(T)$ have The same sign $\pm$

Slowing down = When $a(T)$ and $V(T)$ have different signs $\pm$

9. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

a) Find the velocity of the particle at time t = 2. Indicate units of measure.

$$S'(T) = 3T^2 - 12T + 9 = 3(T^2 - 4T + 3) = 3(T-1)(T-3) = \text{velocity}$$

$$S'(2) = 3(2-1)(2-3) = 3 \cdot 1 \cdot -1 = -3 \text{ m/s}$$

b) When is the particle at rest? Justify your response.

$V(T) = S'(T) = 0$

$S'(T) = V(T) = 3(T-1)(T-3)$ at rest when $T=1$ or $T=3$ because $V(1)=0$, $V(3)=0$

c) Determine whether the particle is speeding up or slowing down at time t = 4. Justify your response.

Need to know $V(4)$ and $a(4) = S''(4) = V'(4)$

$S(T) = T^3 - 6T^2 + 9T$

$S'(T) = V(T) = 3T^2 - 12T + 9 = 3(T-1)(T-3) \Rightarrow V(4) = 3(4-1)(4-3) = 3 \cdot 3 \cdot 1 \Rightarrow$ positive

$S''(T) = a(T) = 6T - 12 = 6(T-2) \Rightarrow a(4) = 6(4-2) = 6 \cdot 2 \Rightarrow$ positive

Speeding up $V(4) = +9$, $a(4) = +12$

Both $V(T)$ and $a(T)$ are positive values at $T=4$. Since $a(T)$ and $V(T)$ are the same sign, the particle is speeding up.

1. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s(t) = 24t - 0.8t^2$ meters in t seconds.

V_0

a) Find the rock's velocity $S'(T)$ and acceleration $S''(T)$ as functions of time. (The acceleration in this case is the acceleration of gravity on the moon).

b) How long did it take the rock to reach its highest point? $V(T) = 0$ Find T, then plug in to $S(T)$ time

c) How high did the rock go?

d) When did the rock reach half its maximum height? $S(T) =$ half the max height. Solve for T

e) How long was the rock aloft? Lands when $S(T) = 0$

"Takes off" when $S(T) = 0 \Rightarrow T = 0$ "take off"